

## Extension of the Complex Torque Coefficient Method for Synchronous Generators to Auxiliary Devices in Electrical Networks

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### Keywords

Generator excitation systems, devices, control, simulation, electrical machines.

### Abstract

The aim of the proposed paper is to extend the well known complex torque coefficient method for synchronous machine rotor oscillations, to auxiliary devices (voltage regulators, power system stabilizers (PSS), ...) in electrical networks. To illustrate the effectiveness of the proposed extension, the study is performed on a model power system. Results obtained by this method are compared with those given, in the one hand, by an eigenvalue analysis, and in the other hand by numerical simulations.

### Introduction

The concept of a complex torque coefficient method for synchronous machine rotor oscillations is well known and has been used in the technical literature in the early 1930's. This concept has been generalized to the torsional interaction [1,2] of the synchronous machine by including electrical and mechanical systems. The aim of this study is to apply and extend this method to other devices in electrical networks, particularly in the present paper to voltage regulators and power system stabilizers. In the expressions of the complex torque coefficient obtained in these cases, a decoupling between the effect due to the electrical system of the machine, and the one due to the auxiliary device has been realized, in order to show separately the damping (or undamping) effect due to the auxiliary device.

### Theoretical study

#### Complex torque coefficient method

A synchronous machine develops an additional torque  $\Delta t_{em}$  in response to small rotor oscillations.  $\Delta t_{em}$  is obtained from the linearization of the electromagnetic torque, and in the Laplace domain, is given by :

$$\Delta t_{em} = [x_d(p) \cdot iq_0 - \Psi q_0] \cdot \Delta id + [\Psi d_0 - x_q(p) \cdot id_0] \cdot \Delta iq + G_f(p) \cdot iq_0 \cdot \Delta u_f \quad (1)$$

$u_f$  is the excitation voltage,  $x_d(p)$ ,  $x_q(p)$  are the operational reactances of the generator given by :

$$x_d(p) = \frac{x_d(1 + pT'd)(1 + pT''d)}{(1 + pT'd_0)(1 + pT''d_0)} \quad (2)$$

$$xq(p) = \frac{xq(1 + pT''q)}{(1 + pT''q0)} \quad (3)$$

$G_f(p)$  is an operator expressed as function of the derivative operator  $p$ , and represents the transfer function of the excitation.

$$Gf(p) = \frac{lad}{rf} \cdot \frac{(1 + pT\sigma D)}{(1 + pT'd0)(1 + pT''d0)} \quad (4)$$

For rotor oscillations of amplitude  $\epsilon$  with frequency  $fr = \lambda \cdot fn$ , one can substitute  $p = j \cdot \lambda \cdot \omega n$ . This yields  $\underline{\Delta tem}$  in the complex form of representation :

$$\underline{\Delta tem} = \underline{Ke}(j \cdot \lambda) \cdot \underline{\epsilon} = (Ke + j \cdot \lambda \cdot De) \cdot \underline{\epsilon} \quad (5)$$

$Ke$  represents the electrical spring constant,  $De$  the electrical damping constant, and  $\underline{\epsilon}$  the generator rotor oscillation.

$Ke$  and  $De$  depend upon the :

- power system configuration
- operation point of the generators
- number of machines operating in parallel, etc.

In the same manner, by introducing a 'complex torque coefficient for the mechanical system  $\underline{Km}(j \cdot \lambda) = (Km + j \cdot \lambda \cdot Dm)$ ', the behaviour of the whole system is described by the following basic equation of oscillation :

$$[(Ke + Km) + j \cdot \lambda \cdot (De + Dm)] \cdot \underline{\epsilon} = 0 \quad (6)$$

By extending this method to the auxiliary devices, a decoupling between the effects due to the electrical and mechanical systems, and to the one due to the auxiliary device has been realized. This leads to the introduction in Equation (6) of a 'supplementary complex coefficient'  $\underline{Ksup}(j \cdot \lambda) = (Ksup + j \cdot \lambda \cdot Dsup)$  defined for each case, according to the device used.

$$[(Ke + Km + Ksup) + j \cdot \lambda \cdot (De + Dm + Dsup)] \cdot \underline{\epsilon} = 0 \quad (7)$$

If there is no damping  $(De + Dm + Dsup) = 0$ , the frequencies of shaft oscillation  $fr = \lambda \cdot fn$  must satisfy the following condition :

$$(Ke + Km + Ksup) = 0 \quad (8)$$

If the resulting damping  $(De + Dm + Dsup)$  is positive, then the oscillation would decay. If the resulting damping for the frequencies satisfying equation (8) is negative, then the system becomes unstable.

### Eigenvalue analysis

This well-known method is based on the linearization of the coupled electrical and mechanical equations under small perturbations.

## System under study

Figure 1 represents the system under study. It consists of three synchronous generators connected to an infinite bus through transmission lines. Two loads C1 and C2 are considered. The voltage regulators RU1 to RU3 are standard regulators of PID types. The corresponding block diagrams are given in Figure 2. The ones of the power system stabilizers PSS1 to PSS3 are given in figure 3.

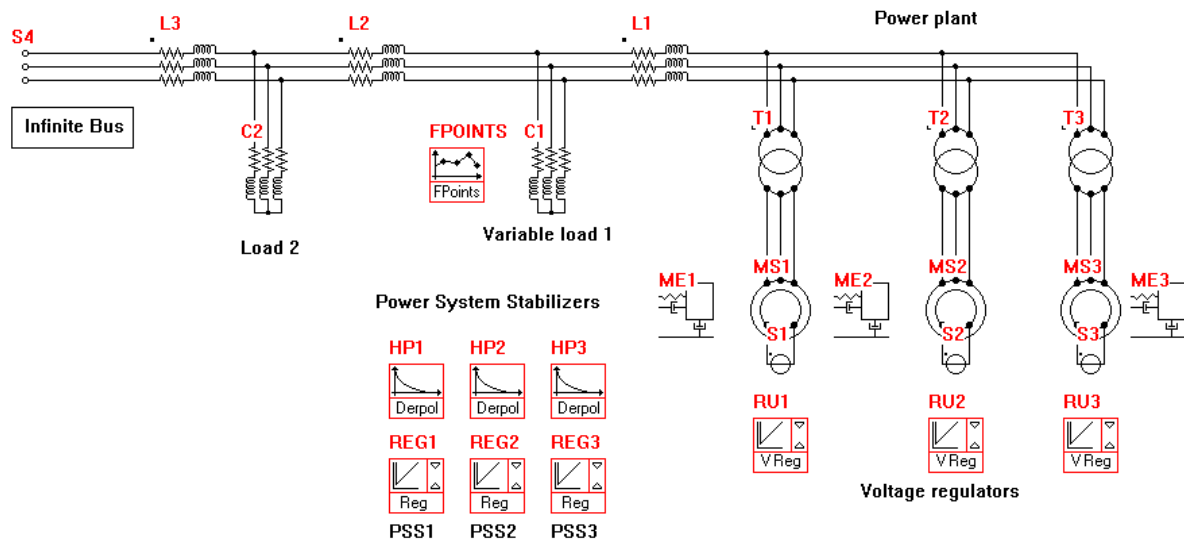


Fig. 1: System under study

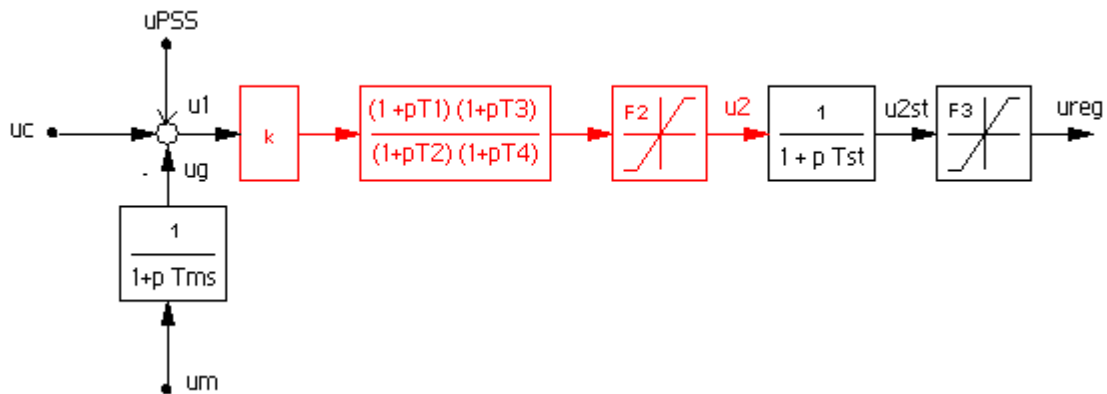


Fig. 2: Block diagram of the voltage regulators RU1 to RU3

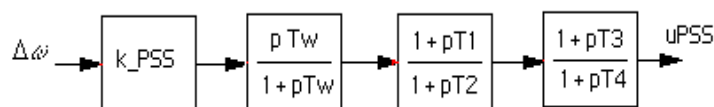


Fig 3: Block diagram of the power system stabilizers PSS1 to PSS3

## Case 1: Study of the effect of variation of the gain of the voltage regulators on the stability of the power plant

### Complex torque coefficient method

#### a) System without voltage regulators and without PSS (number of machines = 3)

The three synchronous generators are identically preloaded at their rated values ( $P = 400$  MW,  $U_N = 21$  kV). Figure 4 represents the coefficients  $K_e$ ,  $K_m$ ,  $K_{tot} = K_e + K_m$  versus the frequency (gain  $k$  of the voltage regulators = 0). Figure 5 represents the coefficients  $D_e$ ,  $D_m$ ,  $D_{tot} = D_e + D_m$  versus the frequency. In this case no mechanical damping is considered ( $D_m = 0$ ).

One notice that  $K_{tot} = 0$  gives the oscillation frequency of the rotor  $f = 0.933$  Hz. The corresponding  $D_{tot}$  is positive, and the system is stable in this case.

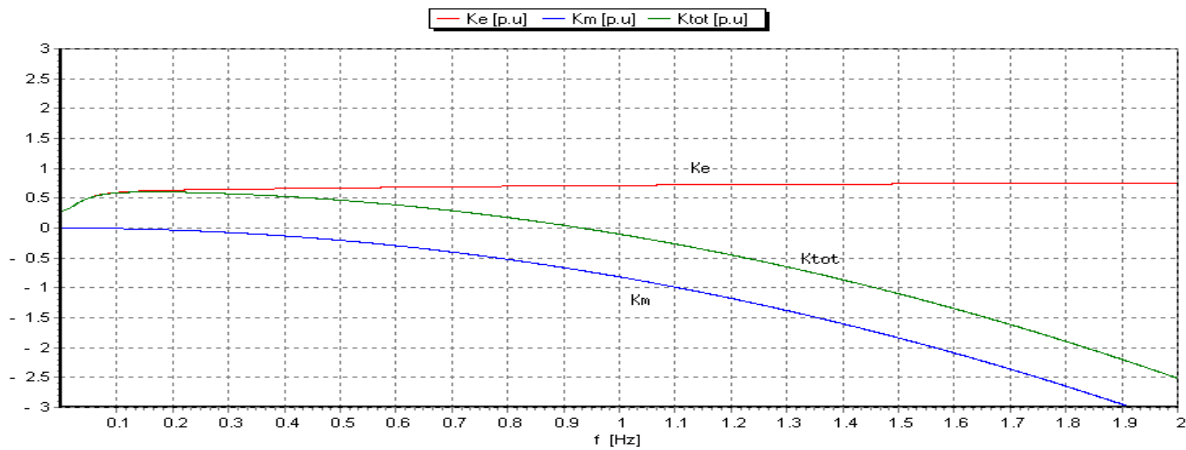


Fig 4:  $K_e$ ,  $K_m$ ,  $K_{tot} = K_e + K_m$  versus the frequency (gain  $k$  of the voltage regulators = 0).

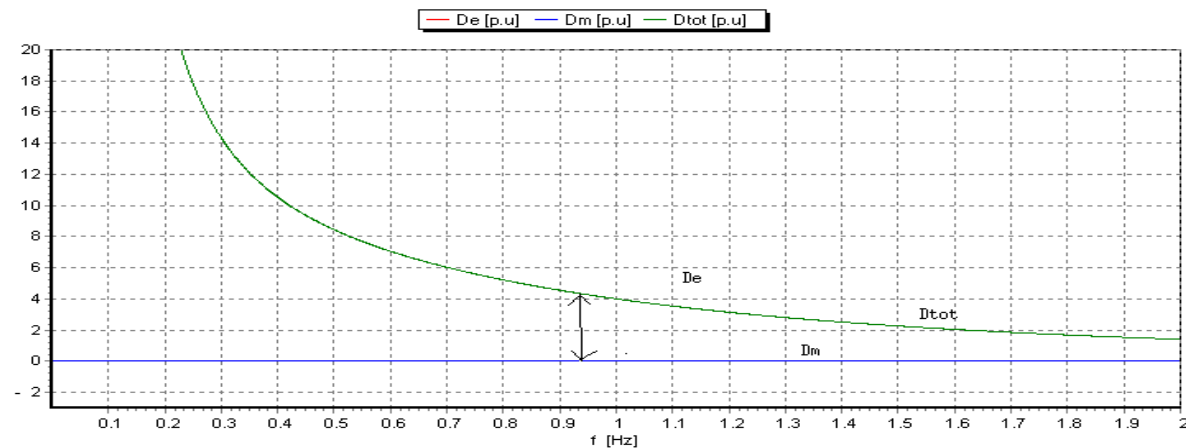


Fig 5:  $D_e$ ,  $D_m$ ,  $D_{tot} = D_e + D_m$  versus the frequency (gain  $k$  of the voltage regulators = 0).

#### b) System with voltage regulators and without PSS (number of machines = 3)

Figures 6 and 7 represent respectively the coefficients  $K_{tot} = K_e + K_m + K_{sup(VR)}$  and  $D_{tot} = D_e + D_m + D_{sup(VR)}$  versus the frequency, with the gain  $k$  of the voltage regulators equal respectively to 0 ; 50 ; 100 ; 127 ; 150.

$K_{sup(VR)}$  and  $D_{sup(VR)}$  are the supplementary coefficients due to the voltage regulators. One notice that  $K_{tot} = 0$  leads to  $f \approx 0.94$  Hz. The resulting damping  $D_{tot}$  becomes negative for a value of the gain  $k$  of the voltage regulators exceeding 127 in this case. Figure 8 shows the undamping (negative damping) introduced by the voltage regulators for different values of the gain  $k$ .

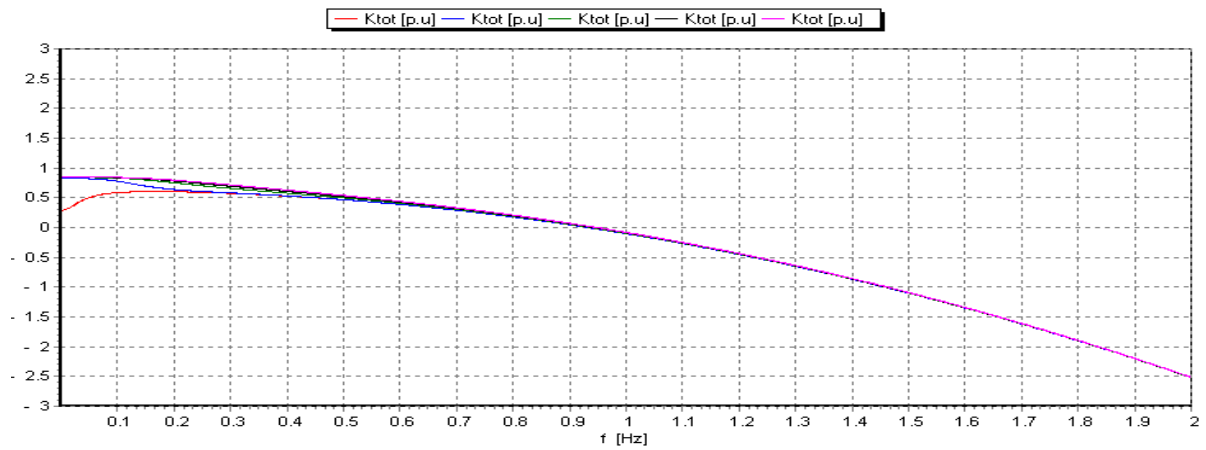


Fig 6:  $K_{tot} = K_e + K_m + K_{sup(VR)}$  versus the frequency.

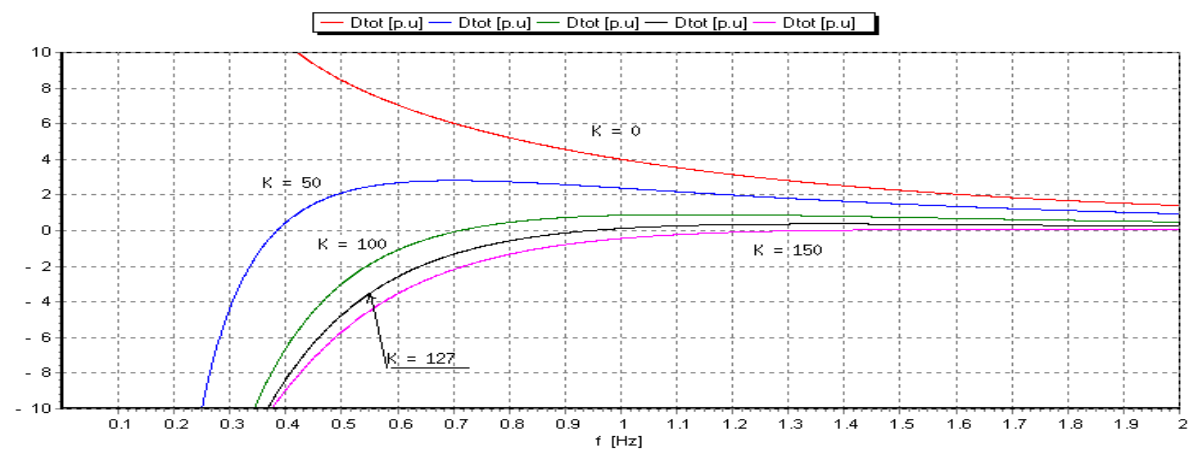


Fig 7:  $D_{tot} = D_e + D_m + D_{sup(VR)}$  versus the frequency, for different values of the gain  $k$  of the voltage regulators ( $D_m = 0$ ).

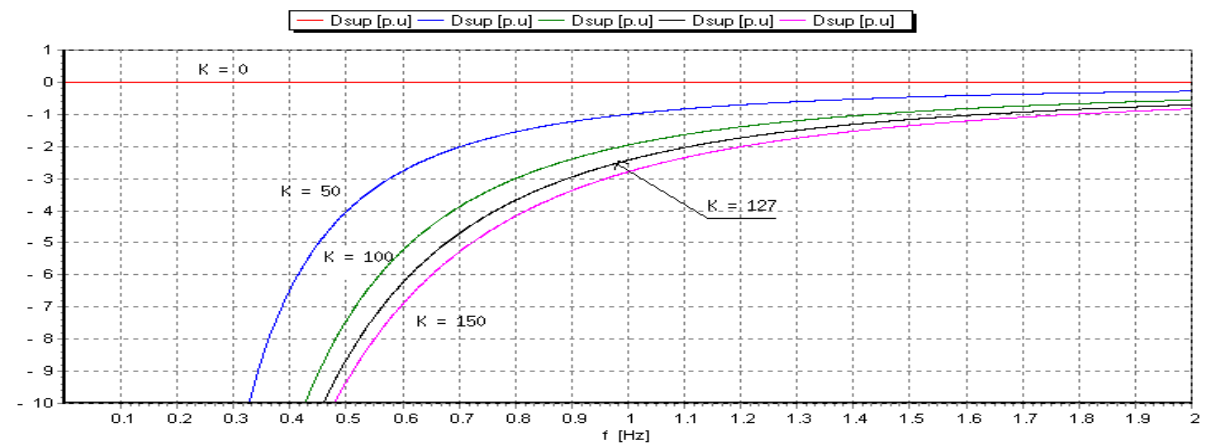


Fig 8:  $D_{sup(VR)}$  versus the frequency, for different values of the gain  $k$  of the voltage regulators.

## Eigenvalue analysis

### a) System without PSS (number of machines = 1, 2 or 3)

Figure 9 represents the real part of the eigenvalue corresponding to the mechanical mode of the generators versus the gain  $k$  of the voltage regulators when one, two or three machines are operating in parallel in the power plant and preloaded at their rated values ( $P = 400$  MW,  $U_N = 21$  kV). In the case of one or two generators operating in parallel in the power plant and preloaded to their rated values, the system remains stable (real part of the eigenvalue corresponding to the mechanical mode  $< 0$ ). In the case of three generators operating in parallel, if the value of the gain  $k$  exceeds 127 in this case, the real part of the eigenvalue corresponding to the mechanical mode is positive, and the system becomes unstable. These results confirm those obtained by the complex torque coefficient method.

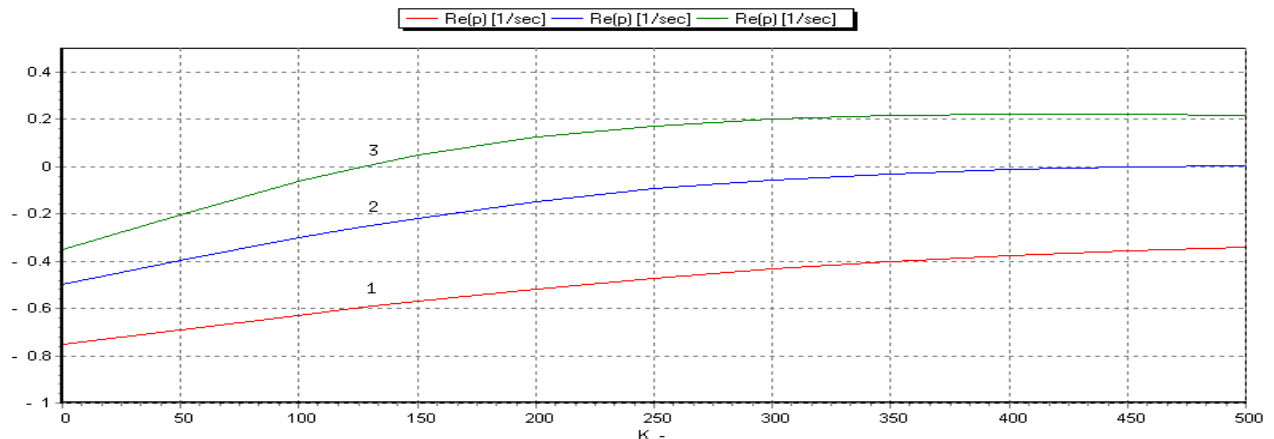


Fig 9: Real part of the eigenvalue corresponding to the mechanical mode of the generators versus the gain  $k$  of the voltage regulators.

- 1: One machine operating in the power plant
- 2: Two machines operating in parallel
- 3: Three machines operating in parallel

## Numerical simulations

### a) System with voltage regulators and without PSS (number of machines = 3)

The system behavior has been simulated using the modular software package SIMSEN [3]. Figures 10 and 11 represent respectively the variation of the active power of the load C1 and the internal angle of the generators for three values of the gain  $k$  of the voltage regulators ( $k = 100$  ;  $127$  ;  $200$ ). One can notice that the results obtained by the three approaches are in very good agreement.

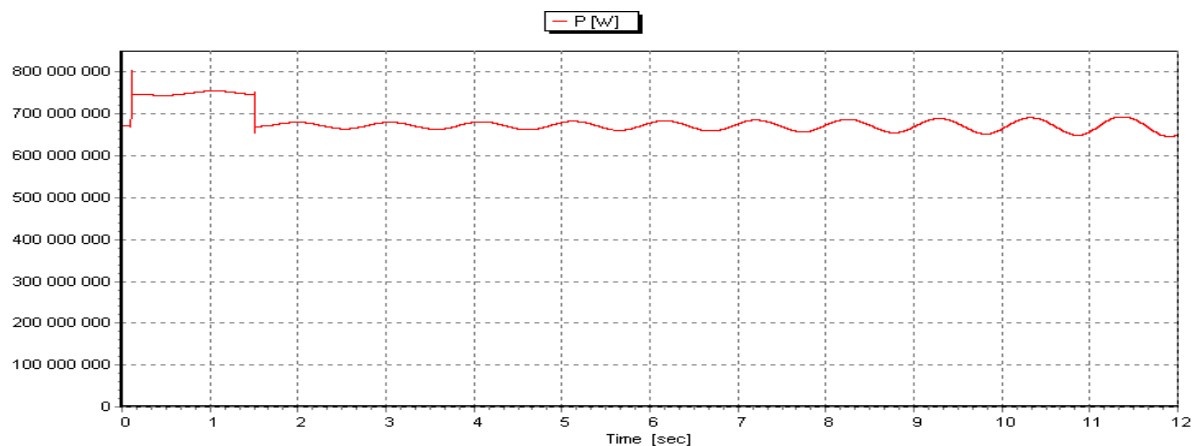


Fig 10: Active power of the variable load C1.

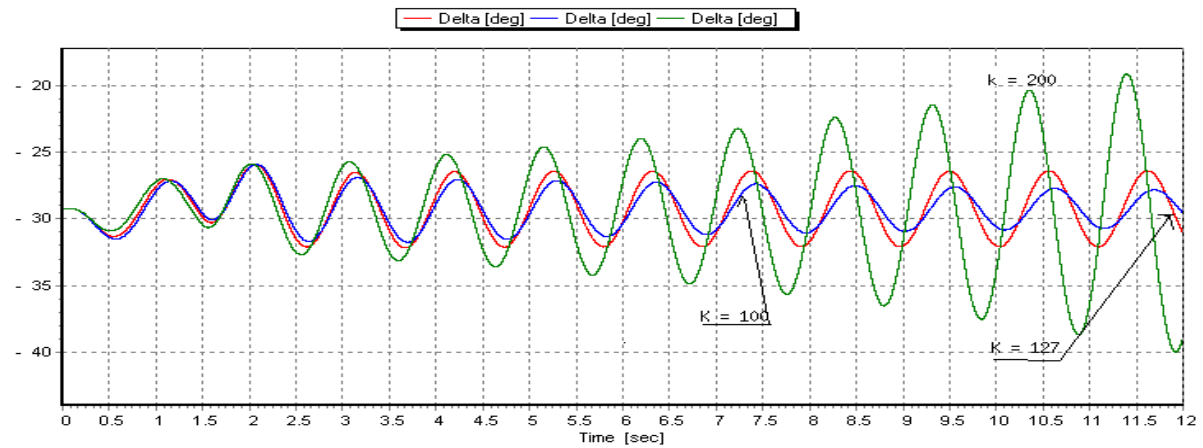


Fig 11: Internal angle of the generators for three values of the gain  $k$  of the voltage regulators ( $k = 100; 127; 200$ )

## Case 2: Study of the effect of the power system stabilizers on the stability of the power plant

### Complex torque coefficient method

#### System with voltage regulators and with PSS (number of machines = 3)

The three synchronous generators are identically preloaded at their rated values ( $P = 400$  MW,  $U_N = 21$  kV). The gain  $k$  of the voltage regulators is equal to 200. Figure 12 represents the coefficient  $K_{tot} = K_e + K_m + K_{sup(VR)} + K_{sup(PSS)}$  versus the frequency with respectively the following values of the gain  $k_{PSS}$  (0 ; 5, 10). Figure 13 shows that for a gain  $k_{PSS} = 5$ , the resulting damping  $D_{tot}$  is positive, and the system becomes stable. One can notice (figure 14) that for the oscillation frequency  $f \cong 0.96$  Hz, the total supplementary damping introduced by the voltage regulators and by the PSS is positive.

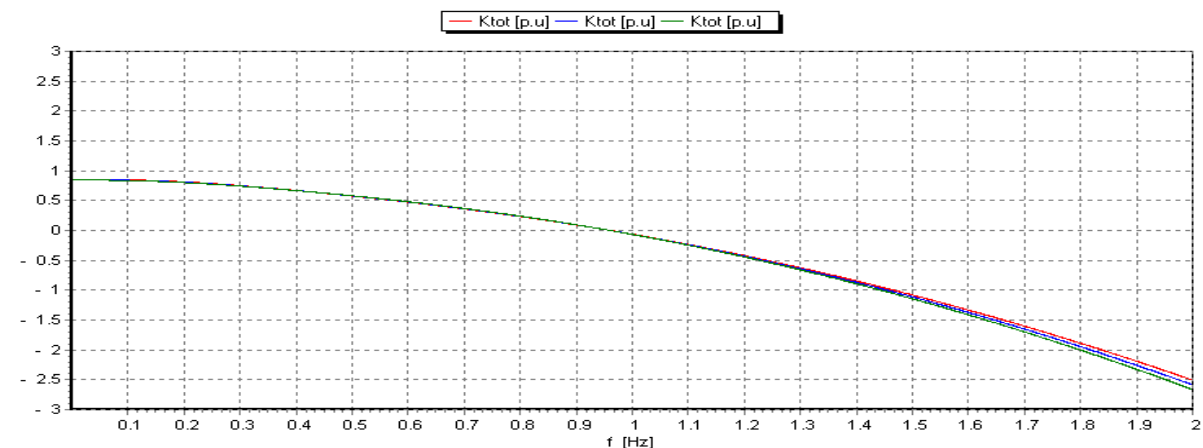


Fig 12:  $K_{tot} = K_e + K_m + K_{sup(VR)} + K_{sup(PSS)}$  versus the frequency with respectively the following values of the gain  $k_{PSS}$  (0 ; 5; 10). The gain  $k$  of the voltage regulator is equal to 200.

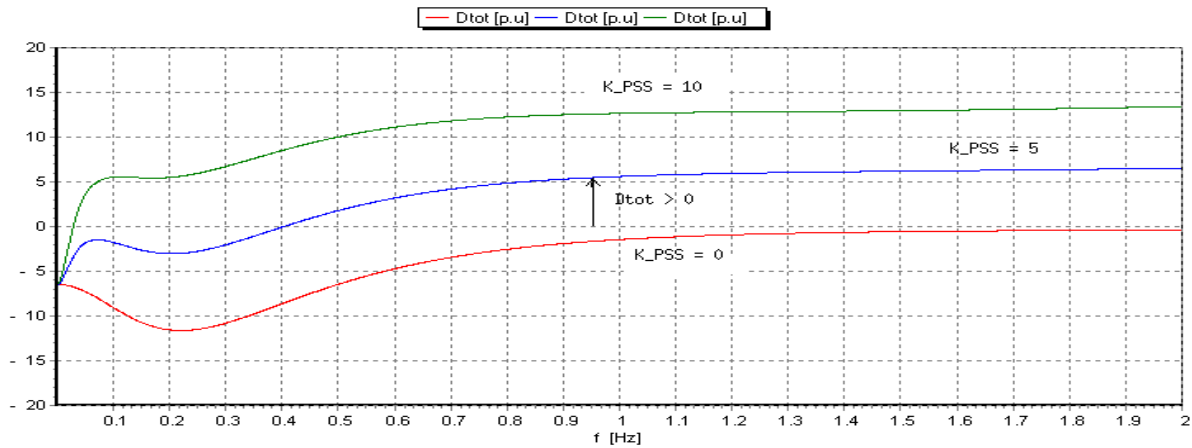


Fig 13:  $D_{tot} = D_e + D_m + D_{sup(VR)} + D_{sup(PSS)}$  versus the frequency ( $D_m = 0$ ). The gain  $k$  of the voltage regulator is equal to 200.

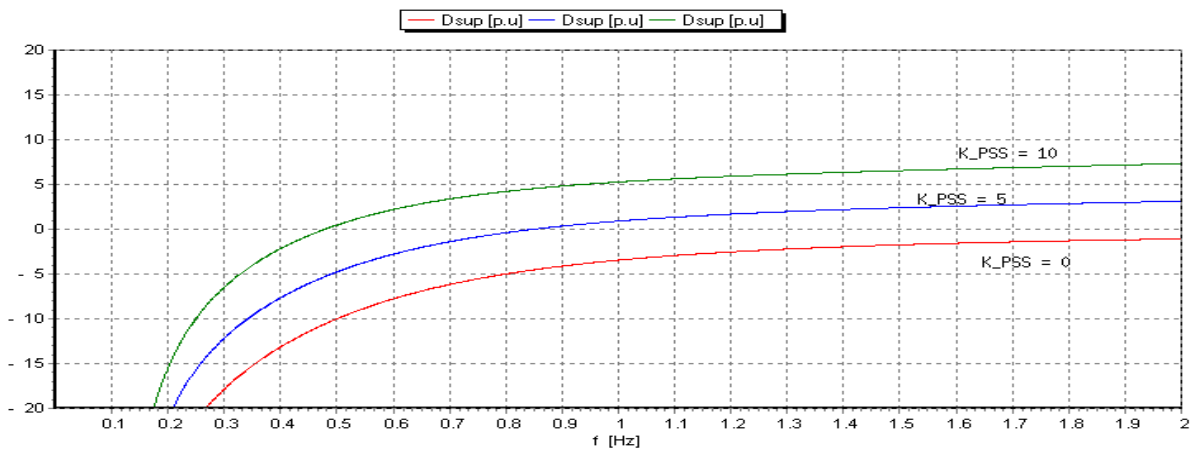


Fig 14: Total supplementary damping  $D_{sup} = D_{sup(VR)} + D_{sup(PSS)}$  versus the frequency.

## Eigenvalue analysis

### System with voltage regulators and with PSS (number of machines = 3)

The gain  $k$  of the voltage regulators is equal to 200. Figure 15 shows that a gain  $k_{PSS}$  of the power system stabilizers exceeding 1.2 leads to a stable system (real part of the eigenvalue corresponding to the mechanical mode  $< 0$ ).

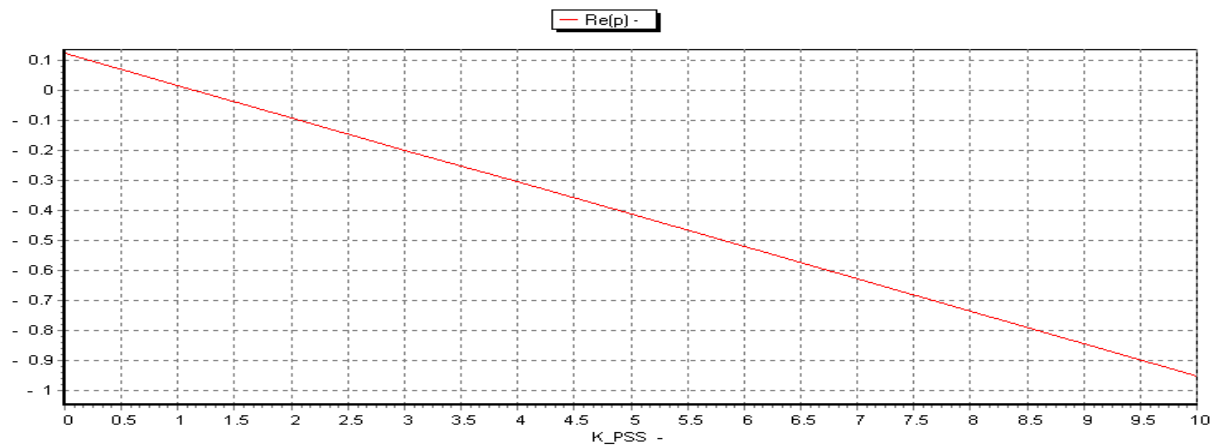


Fig 15: Real part of the eigenvalue corresponding to the mechanical mode of the generators versus the gain  $k_{PSS}$  of the power system stabilizers.



## Numerical simulations

### System with voltage regulators and with PSS (number of machines = 3)

The gain  $k$  of the voltage regulators is equal to 200. Figure 16 shows that for a gain  $k_{PSS}$  of the power system stabilizers equal to 5 the system behavior is stable.

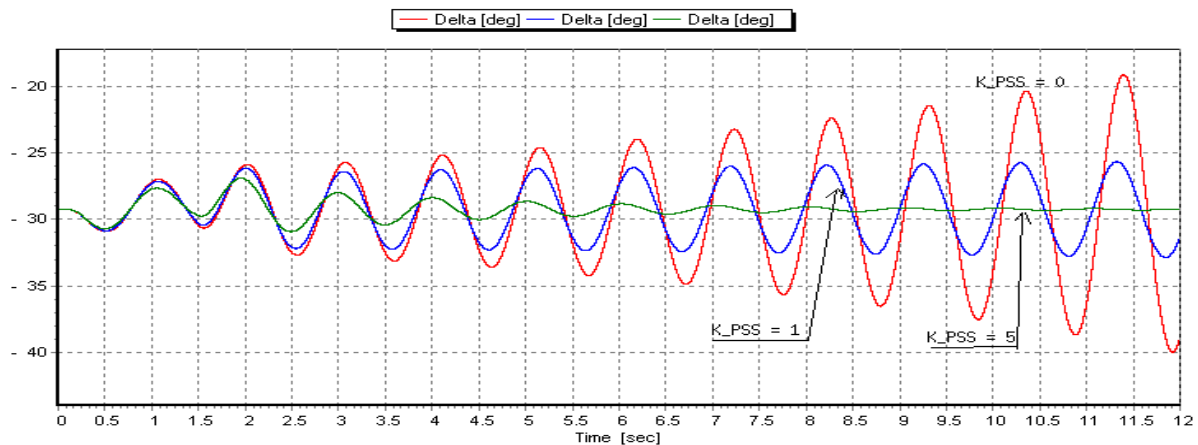


Fig 16: Internal angle of the generators for three values of the gain  $k_{PSS}$  of the power system stabilizers ( $k_{PSS} = 0; 1; 5$ )

## Conclusion

An extension of the complex torque coefficient method for synchronous generators to auxiliary devices (voltage regulators and power system stabilizers) in electrical networks is proposed. In the analytical expressions developed for the basic equation of oscillation, and for each device, a supplementary complex coefficient due to the device effect is calculated separately. Different results are presented in order to show the effectiveness of the proposed method. These results are compared with those obtained by an eigenvalue analysis and by numerical simulations. This powerful method which can be applied in a wide range of frequency (0, 2 fn), can be extended to other devices in the electrical networks (SVC Static Var compensators, phase shifters...) and used also for the analysis of subsynchronous resonance in networks with series capacitors.

## References

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## Appendix

### Machine parameters

Rated active power: 400 MW  
Frequency: 50 Hz  
Rated voltage: 21 kV

### Voltage regulators parameters

$k = 200$ ;  $T_{ms} = 0.04$ ;  $T_1 = 0.1$ ;  $T_2 = 0.02$ ;  
 $T_3 = 1$ ;  $T_4 = 5$ ;  $T_{st} = 0.005$

### PSS parameters

$k_{PSS} = 5$ ;  $T_w = 5$ ;  $T_1 = 0.1$ ;  $T_2 = 0.025$ ;  
 $T_3 = 0.33$ ;  $T_4 = 0.18$